

# ANGULAR MOTION



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## UNITS OF MEASURE:

**angular velocity ( $\omega$ )** -  $^{\circ}/s$  (degrees/second)

*like velocity ( $v$ )* -  $m/s$

**angular acceleration ( $\alpha$ )** -  $^{\circ}/s^2$

*like acceleration ( $a$ )* -  $m/s^2$

# NEWTON'S 3 LAWS APPLIED TO 'ANGULAR MOTION'

## **EQUAL AND OPPOSITE LAW:**

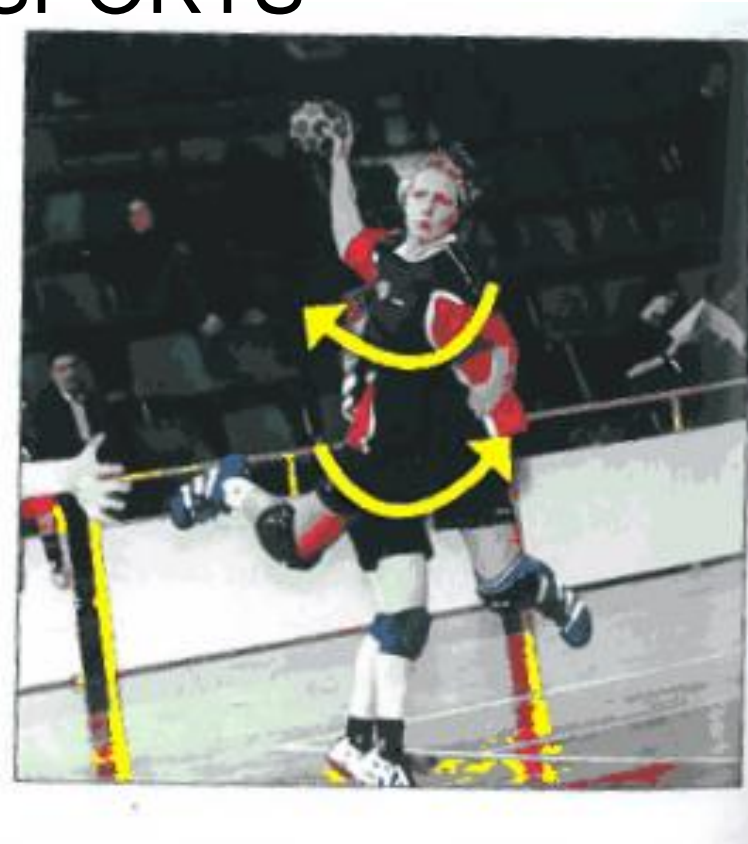
“For every TORQUE that is exerted by one body onto another, there is an equal and opposite TORQUE exerted as well.”

**TORQUE - force causing angular motion (Newtons)**

Ex. Sitting on a spinning chair: swinging your arms horizontally in one direction, will make you....

# NEWTON'S LAW OF EQUAL & OPPOSITE TORQUE APPLIED TO SPORTS

Effective application of the action–reaction principle is also made in throwing and kicking. In handball, for example, the throwing action is carried out by moving the shoulder forward along with the throwing arm – action. To prevent the whole body from twisting, which is important for good throwing accuracy, the athlete twists the hips forward in the opposite direction – reaction. This also engages the powerful trunk muscles, which significantly increases the power of the throw.



# NEWTON'S 3 LAWS APPLIED TO 'ANGULAR MOTION'

## **FORCE AND ACCELERATION LAW:**

“The angular acceleration of an object is proportional to, and in the same direction as the ‘TORQUE’ which is applied to it.”

**again...TORQUE = force causing angular motion**

Recall:  $F = m \times a$  In angular world there's a little more to it...

Which skater is spinning faster?



# NEWTON'S 3 LAWS APPLIED TO 'ANGULAR MOTION'

Linear motion:

$$F = m \times a$$

Angular Motion:

$$T = I \times \alpha$$



**TORQUE = 'Moment of Inertia' x angular acceleration**

If **mass (m)** is what makes an object hard to push....

it is '**Moment of Inertia' (I)** that makes an object hard to spin

**MOMENT OF INERTIA** - "that characteristic of an object which makes it reluctant to change its angular motion"

- a factor of the object's mass and radius

$$I = m \times r^2$$

Example....spinning on a chair - arms in / arms out  
- does mass change? does radius?

IMPORTANT: in the formula  $I = m \times r^2$   
...why is radius squared?

...because RADIUS MATTERS MORE!!!

- if you increase the mass of an object, it requires more Torque to spin it,
- but if you increase its radius, it requires A LOT more Torque to spin it!!!

$$T = I \times \alpha$$

or...

$$T = m \times r^2 \times \alpha$$



# For example: a baseball bat

If you tried to swing a heavier bat, it would be hard to swing

$$\uparrow T = \uparrow m \times r^2 \times \alpha$$

But if you tried to swing a longer bat (even if it was the same mass), it would be waaaaaay harder to swing!

$$\uparrow T = m \times \uparrow r^2 \times \alpha$$



# Newton's Force - acceleration law APPLIED TO 'ANGULAR MOTION'

Ex: If it takes 20 N of Torque to spin a regular bat,  
how much Torque would it require to spin...

a. a bat that is twice as heavy?

$$T = m \times r^2 \times \alpha$$

2x   $T = 2xm \times r^2 \times \alpha$

$$T = 2 \times 20 \text{ N} = 40 \text{ N}$$

b. a bat that is twice as long? (twice the radius)

$$T = m \times 2r^2 \times \alpha$$

$$4x T = m \times 4r \times \alpha$$

$$T = 4 \times 20 \text{ N} = 80 \text{ N}$$

or, try that same question with real numbers...

How much Torque would it take to swing a bat that weighs...

- a) 2 kg and has a radius of 0.7m?
- b) 4 kg and has a radius of 0.7m?
- c) 2 kg and has a radius of 1.4m?

Answers:

- a) 0.98 N
- b) 1.96 N (twice as much Torque required)
- c) 3.92 N (4 times as much Torque required)

# NEWTON'S 3 LAWS APPLIED TO 'ANGULAR MOTION'

## LAW OF INERTIA:

"A rotating object will continue to spin with constant angular momentum, unless an external force acts upon it."

Recall:

**Momentum:**

$$M = m \times v$$

**Angular Momentum (H):**

$$H = \underbrace{m \times r^2}_{\text{(Moment of Inertia)}} \times \omega$$

(Moment of Inertia)

\*Important concept: In free-fall Angular Momentum (H) is held CONSTANT. It does not change (see Law of Inertia). Likewise, in all future spinning examples H will be assumed to be constant, (ie. we are ignoring forces like friction, air resistance, etc.) In other words:  **$H_{\text{initial}} = H_{\text{final}}$**

# The LAW of inertia (CONT'D)

Therefore, if an object is spinning...  
any decrease in Moment of Inertia (mass or radius) will  
result in an increase in angular velocity ( $\omega$ ) BECAUSE **H**  
**IS CONSTANT!!!**

$$H = \downarrow m \times r^2 \times \uparrow \omega$$

and vice versa

Large  $I$   
Small  $\omega$



Small  $I$   
Large  $\omega$



The question is, how  
do you calculate  
these  
changes?

Example: A diver is doing a layout, then pikes herself into ½ her original length (aka radius). How will her angular velocity be affected?



$$H = m \times r^2 \times \omega$$

$$H = m \times \downarrow r^2 \times \uparrow \omega$$

$$H = m \times \frac{1}{2}r^2 \times \text{___} \omega$$

$$H = m \times \frac{1}{4} \times \underline{4} \omega$$

(H is constant)  $H_{\text{initial}} = H_{\text{final}}$

Therefore, her angular velocity will be 4 times greater if she pikes to ½ her original radius.